

# Pion-nucleon scattering in the $K$ -matrix approach

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**Abstract.** The  $K$ -matrix approach with effective Lagrangians is used to describe the  $S$  and  $P$  pion-nucleon partial-wave amplitudes in the energy range  $E_{lab} \leq 1$  GeV. It is demonstrated, that treating the resonance as  $K$ -matrix a pole gives the natural way to separate the resonance and non-resonance parts of the  $\pi N$  amplitude. The model includes all the four-star  $\pi N$  resonances, the non-resonance contributions are calculated from relevant Feynman graphs without any phenomenological form factors. Different contributions to the inelastic  $\pi^- p \rightarrow \eta n$  amplitude are estimated.

**PACS.** 14.20.Gk Baryon resonances – 13.75.Eg Partial wave analysis

## 1 Introduction

In recent years, there has been a considerable interest, both theoretical and experimental, in the study of the  $\pi N$  interaction at intermediate energies. The reason (for this) was to obtain a rich information contained in such data. At low energies, the  $\pi N$  scattering is one of the best tools for testing small violations of the chiral symmetry and isotopic invariance. At higher energies a set of resonances was found in the  $\pi N$  system. The masses and branching ratios of these resonances can be used as a test of the quark structure of baryons. The important problem is to extract these parameters from experimental data. The usual way to do this is to use results of the phase shift analysis. Later on some resonance parameters published in PDG [1] were obtained using the Dalitz-Tuan representation [2] for the phase shifts  $\delta$ . The basic ansatz for this representation is that the phase shifts  $\delta_r$  coming from resonance (a Breit-Wigner type formula is applied here) and background  $\delta_b$  are summed. In this case, the  $S$ -matrix for pure elastic scattering has a form:

$$S(w) = e^{2i\delta_b(w)} \frac{M - w + i\frac{\Gamma}{2}}{M - w - i\frac{\Gamma}{2}}, \quad (1)$$

where  $w$  is the total c.m. energy,  $M$  is mass of the resonance and  $\Gamma$  is its width.

This prescription comes only from the unitarity, and nothing tells us about the origin and energy dependence of the background. Moreover, the prescription is not unique. Bofinger and Woolcock [3] demonstrated that different prescriptions might be described as members of the following one-parameter family (see also [4]):

$$\tan \delta = \frac{\tan \delta_b + \tan \delta_r}{1 + \lambda \tan \delta_b \tan \delta_r}, \quad (2)$$

the parameter  $\lambda$  being arbitrary. The  $K$ -matrix approach to  $\pi N$  scattering [5] gives a rather natural argument for an adequate choice of the  $\lambda$  value and a way to estimate the background. It was demonstrated in [5,6] that this approach leads to a good description of all the  $\pi N$  observables in the whole elastic region including  $\Delta_{33}$  resonance. The main objective of this paper is to study the energy interval with open inelastic channels and estimate the possibility to determine resonance parameters from the phase shift analysis using the results of [5] as a background.

## 2 Multichannel $K$ -matrix unitarization procedure

The scattering amplitude obeys the system of Bethe-Salpeter equations having the following form

$$F_{ab}(q_a, q_b, w) = V_{ab}(q_a, q_b, w) + \sum_c \int V_{ac}(q_a, q, w) G_c(q, w) \times F_{cb}(q, q_b, w) dq, \quad (3)$$

the sum in the rhs runs over all open channels,  $G_c$  is the propagator of the intermediate state  $c$ . The system (3) is equivalent to the integral system for the  $K$ -matrix

$$K_{ab}(q_a, q_b, w) = V_{ab}(q_a, q_b, w) + \sum_c \int V_{ac}(q_a, q, w) \text{Re} G_c(q, w) \times K_{cb}(q, q_b, w) dq \quad (4)$$

and to the algebraic one connecting the scattering amplitude with the  $K$ -matrix

$$F_{ab}(q_a, q_b, w) = K_{ab}(q_a, q_b, w) + i \sum_c K_{ac}(q_a, q_c, w) \times q_c F_{cb}(q_c, q_b, w). \quad (5)$$

As is seen from (3) and (4), the  $K$ -matrix includes the same Feynman diagrams as those for the scattering amplitude but without on-mass-shell contribution of intermediate states. Let us consider the case when the leading term of (4) has a pole

$$V_{ab}(q_a, q_b, w) = V_{ab}^B(q_a, q_b, w) + \frac{\alpha_a^0(q_a)\alpha_b^0(q_b)}{M_0 - w}, \quad (6)$$

$M_0$  is the bare mass of the resonance and  $\alpha_{a,b}^0$  are bare vertices. Introducing the background  $K$ -matrix by the equation

$$K_{ab}^B(q_a, q_b, w) = V_{ab}^B(q_a, q_b, w) + \sum_c \int V_{ac}^B(q_a, q_b, w) \text{Re}G_c(q, w) \times K_{cb}^B(q, q_b, w) dq, \quad (7)$$

we get general solution of (4):

$$K_{ab}(q_a, q_b, w) = K_{ab}^B(q_a, q_b, w) + \frac{\alpha_a(q_a, w)\alpha_b(q_b, w)}{M(w) - w} \quad (8)$$

$$\alpha_0(q_a, w) = \alpha_a^0(q_a) + \sum_c \int K_{ac}^B(q_a, q, w) \text{Re}G_c(q, w) \alpha_c^0(q) dq \quad (9)$$

$$M(w) = M_0 - \sum_c \int \alpha_c^0(q) \text{Re}G_c(q, w) \alpha_c(q, w) dq. \quad (10)$$

The energy dependence of the quantity  $M(w)$  can be taken into account by renormalization of the vertex. Indeed, the mass of the resonance  $M_R$  (now we define it as a  $K$ -matrix pole) is the solution of the equation

$$M(w) - w = 0, \quad (11)$$

the quantity  $M(w)$  thus being of the form

$$M(w) = M_R + B(w)(M_R - w). \quad (12)$$

Putting (11) into (7), we obtain finally:

$$K_{ab}(q_a, q_b, w) = K_{ab}^B(q_a, q_b, w) + \frac{\xi_a(q_a, w)\xi_b(q_b, w)}{M_R - w} \quad (13)$$

$$\xi_a(q_a, w) = \alpha_a(q_a, w)[1 + B(w)]^{-1/2}. \quad (14)$$

(13) gives a natural way for splitting the  $K$ -matrix into pole and non-pole terms and defining the resonance as a  $K$ -matrix pole. In this case, the background is the sum of all Feynman diagrams for the  $K$ -matrix without pole

in the intermediate state. For pure elastic scattering (all inelastic channels closed) the scattering amplitude

$$F = \frac{K}{1 - iqK} = \frac{\xi(q)\xi(q) + K^B(q)(M_R - w)}{M_R - w - iq[\xi(q)\xi(q) + K^B(q)(M_R - w)]} \quad (15)$$

have the Breit-Wigner form near the resonance position  $w = M_R$ . In this case, the  $K$ -matrix unitarization procedure corresponds to  $\lambda = 0$  in (2). As is seen from (15), the background contribution to the amplitude vanishes at  $w = M_R$ . Thus, the elastic resonance position corresponds to the energy, for which  $\delta = \pi/2$  even if the background is present. The multichannel case is more complicated. In the absence of the background the scattering amplitude

$$F_{ab}(q_a, q_b, w) = \frac{\xi_a(q_a, w)\xi_b(q_b, w)}{M_R - w - i \sum_c q_c \xi_c^2(q_c, w)} \quad (16)$$

still has the multichannel Breit-Wigner form with the total width

$$\Gamma^{tot} = 2 \lim_{w \rightarrow M_R} \sum_c q_c \xi_c^2(q_c, w). \quad (17)$$

But if the background is present, the solution of (5) leads to a complicated function for the scattering amplitude not similar to the Breit-Wigner form. This situation can be improved by using the eigenchannel representation. This means that we define the new channel basis ( $\alpha, \beta, \gamma \dots$  instead of  $a, b, c \dots$  in (5)) to transform the  $K$ -matrix (13) into the diagonal form  $K^e$

$$K^e = U^+ K U, \quad (18)$$

where  $U$  is the unitary transformation matrix. As is seen from (13), the matrix of amplitudes becomes diagonal too. Only one channel contains the resonance in this representation (see Appendix for details) with the  $K$ -matrix element

$$K_R = \sum_a \frac{\xi_a(q_a, w)\xi_a(q_a, w)}{M_R - w} + \varphi(w), \quad (19)$$

where  $\xi_a(q_a, w)$  are the same quantities as in (13) and  $\varphi(w)$  is the non-resonance part. The eigenchannels are independent (there are no transitions between them), hence the amplitude for the resonance channel has the same form (15) as for pure elastic scattering with the width (17). Then branching ratios can be calculated using the transformation matrix  $U$ . This situation is not new for quantum mechanics. Consider as an example the  $\pi^-p$  scattering. The measurable channels are  $\pi^-p \rightarrow \pi^-p$ ,  $\pi^-p \rightarrow \pi^0n$ ,  $\pi^0n \rightarrow \pi^0n$ . So the scattering amplitude is the  $2 \times 2$  matrix. But the conserved quantity is isotopic spin. Therefore, after transforming this matrix from the charged channel basis to isotopic one, we get the  $2 \times 2$  diagonal matrix. If there is a resonance in this system, it has the conserved isotopic quantum number, so only one channel contains the resonance (for instance  $\Delta_{33}$ ). The branching ratios for the decay of this resonance to the charged states are determined by transformation matrix, i.e. Clebsch-Gordan coefficients. The above arguments show that the parametrization of a resonance as a  $K$ -matrix pole is unambiguous,

and it satisfies all criteria published in [7] for the existence of the resonance.

Another set of resonance parameters is given by the location of the complex pole of the scattering amplitude (see [7,8] for details). The advantage of the  $K$ -matrix parametrization is that quantities in the physical region are involved only. But in any specific analysis both set of the parameters can be calculated.

### 3 A tree-level model for the $K$ -matrix

The quantity  $V$  in (3) is defined as a sum of all irreducible diagrams. There is an infinite number of those, and no way to sum them is known, so  $V$  must be approximated by the finite number of effective diagrams which we believe to be important physically. Let us start with the elastic scattering. Using general principles, such as parity, angular momentum and isospin conservation, the irreducible block  $V$  can be considered as a sum of the  $t$ -channel diagrams with the scalar-isoscalar ( $\sigma$  meson) and vector-isovector ( $\rho$  meson) exchange and  $s$  and  $u$ -channel diagrams with nucleon and the nucleon resonances in the intermediate state. The interaction vertices represent a number of diagrams including the loops coming from closed channels. These loops lead to the finite range of the interaction, therefore the vertices are functions of momenta of entering particles. Such interaction is usually considered within an effective Lagrangian approach, and phenomenological form factors are introduced to account for the loop contributions. This way was used in [9,10]. The disadvantage is a large number of free parameters together with the ambiguity in reducing the four-dimensional Bethe-Salpeter equation to the three-dimensional one, which can be really solved [11]. In other papers [12,13] the second term in rhs of (4) has been neglected. Still, the use of form factors seems to be inconsistent. Indeed, like form factors, the neglected term corresponds to the loop diagrams also. Moreover, these diagrams give the largest interaction range, because they arise from open channels and, therefore, the lightest particles are involved. On the other hand, the same arguments which were used to construct irreducible block  $V$  can be applied to the  $K$ -matrix itself. So, we can suppose that the  $K$ -matrix, being a solution of (4) has a structure similar to  $V$ , but with different vertices. We do not introduce any phenomenological form factor, because it must arise from (4), thus saving the number of free parameters. Instead, we assume all vertices to be constant in some restricted energy interval and check this assumption by fitting the data. It was demonstrated in [5], that such a tree-level model for the  $K$ -matrix leads to a good description of data in the elastic energy region. In this work we also check it for inelastic processes as well. For  $P_{lab} < 1$  GeV/c, there are only three dominant channels in the  $\pi N$  interaction:  $\pi N \rightarrow \pi N$ ,  $\pi N \rightarrow \eta N$  and  $\pi N \rightarrow \pi \pi N$ .

For the elastic channel, we take into account the same diagrams and Lagrangian densities as in [5] including contribution from all four-star  $S$  and  $P$   $\pi N$  resonances. These resonances play an important role at intermediate

energies, therefore the pseudoscalar-pseudovector mixing in  $\pi NN^*$  interaction is additionally assumed. For  $S11$  resonances:

$$L_{\pi NN^*} = -\frac{g_{\pi NN^*}}{1 + X_{\pi N^*}} \bar{\Psi}_{N^*} \vec{\tau} \times \left( iX_{\pi N^*} \vec{\pi} - \frac{1}{M_{N^*} - m} \gamma_\mu \partial^\mu \vec{\pi} \right) \Psi_n + h.c. \quad (20)$$

For  $P11$  resonances:

$$L_{\pi NN^*} = -\frac{g_{\pi NN^*}}{1 + X_{\pi N^*}} \bar{\Psi}_{N^*} \gamma_5 \vec{\tau} \times \left( iX_{\pi N^*} \vec{\pi} + \frac{1}{M_{N^*} + m} \gamma_\mu \partial^\mu \vec{\pi} \right) \Psi_n + h.c. \quad (21)$$

For  $S31$  and  $P31$  resonances, we use the same formulae, with  $\vec{\tau}$  being the  $T_{3/2} \rightarrow T_{1/2}$  transition operator. Only pseudovector coupling is possible for the  $3/2$ -spin resonances. In this case:

$$L_{\pi NN^*} = \frac{g_{\pi NN^*}}{M_{N^*} + m} \bar{\Psi}_{N^*}^\mu \theta_{\mu\nu} \vec{\tau} \Psi_N \partial^\nu + h.c. \quad (22)$$

$$\theta_{\mu\nu} = g_{\mu\nu} - \left( Z_{N^*} + \frac{1}{2} \right) \gamma_\mu \gamma_\nu. \quad (23)$$

For the  $\eta$  production, the resonance mechanism and  $t$ -channel  $a_0$  meson exchange [12] are accounted for in an analogy with the  $\pi N$  charge exchange reaction, where the  $t$ -channel  $\rho$  exchange was found to be important [7]. Corresponding Lagrangian densities are:

$$L_{\eta NN^*} = -\frac{g_{\eta NN^*}}{1 + X_{\eta N^*}} \bar{\Psi}_{N^*} \times \left( iX_{\eta N^*} - \frac{1}{M_{N^*} - m} \gamma_\mu \partial^\mu \right) \eta \Psi_n + h.c. \quad (24)$$

for  $S$  resonances and

$$L_{\eta NN^*} = -\frac{g_{\eta NN^*}}{1 + X_{\eta N^*}} \bar{\Psi}_{N^*} \gamma_5 \times \left( iX_{\eta N^*} + \frac{1}{M_{N^*} + m} \gamma_\mu \partial^\mu \right) \eta \Psi_n + h.c. \quad (25)$$

for  $P$  resonances. For the  $t$ -channel  $a_0$  meson exchange

$$L_{a_0 NN} = q_{a_0 NN} \bar{\Psi} \vec{\tau} \Psi \vec{a}_0 \quad (26)$$

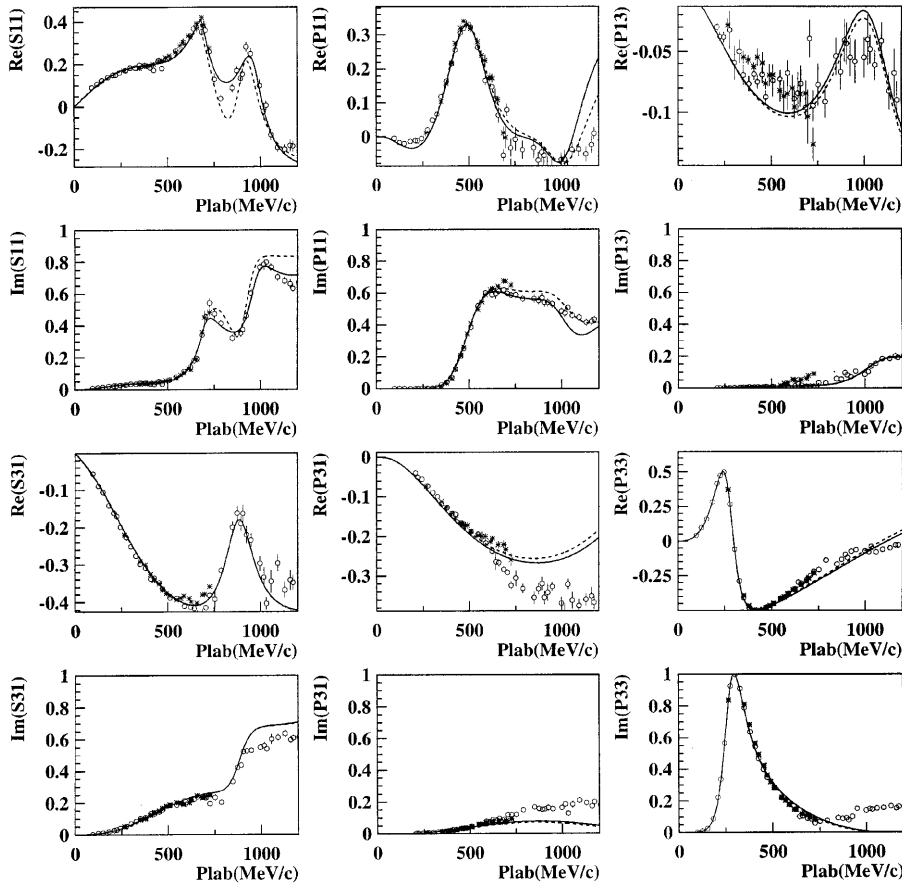
$$L_{a_0 \pi \eta} = m_\eta g_{a_0 \pi \eta} \vec{a}_0 \vec{\pi} \eta. \quad (27)$$

In order to keep the model as simple as possible, for the two pion production we follow a phenomenological quasi-two-body approach [13]. The two pion decay is parametrized by the coupling to the scalar-isoscalar meson  $\phi$  for  $T = 1/2$  resonances and to the vector-isovector meson  $\xi$  for  $T = 3/2$  resonances. The interaction Lagrangians are:

$$L_{\phi NN^*} = g_{\phi NN^*} \bar{\Psi}_{N^*} \gamma_\mu \Psi \partial^\mu \phi + h.c. \quad (28)$$

$$L_{\xi N^* N} = -g_{\xi N^* N} \gamma_\mu \vec{\tau} \Psi \vec{\xi}^\mu + h.c. \quad (29)$$

for  $P$  resonances. For  $S$  resonances the same formulae (28,29) with an extra  $\gamma_5$  matrix are used. We do not fix masses of these  $\phi$  and  $\xi$  dummy particles, instead vertex functions are defined [13] as:



**Fig. 1.** Partial wave amplitudes. Solid line corresponds to the full calculation, dashed line stands for calculation without  $a_0$  exchange,  $\circ$  – PWA by Arndt et al. [14],  $*$  – PWA by Abaev and Kruglov [15]

$$f_{\sigma,v}(s) = \frac{1}{\pi} \int ds_1 \frac{f_{\sigma,v}(s, s_1) M_{\sigma,v} \Gamma_{\sigma,v}}{(s_1 - M_{\sigma,v}^2)^2 + M_{\sigma,v}^2 \Gamma_{\sigma,v}^2}, \quad (30)$$

where  $f_{\sigma,v}(s, s_1)$  is the vertex calculated with the interaction (28,29) for the total two-pion energy squared  $s_1$ . The parameters  $M_{\sigma,v}$  and  $\Gamma_{\sigma,v}$  are chose to give the mass and width of the  $\sigma$  meson and  $\rho$  meson. It was found that the results are not sensitive to precise values of the  $\sigma$ -meson mass and width, so  $M_\sigma = 800$  MeV and  $\Gamma = 400$  MeV [1] were used in the calculation.

## 4 Fitting procedure

Our goal is not to describe the data in details but to demonstrate the validity of the approach and to examine different kinds of couplings and mechanisms for the  $\eta$  meson production. Accordingly, the experimental data used in the fit are the single-energy solutions by Arndt et al. [14] and Abaev and Kruglov [15] for  $S$  and  $P$  partial wave amplitudes. We also try to fit the KH80 [7] solution. For the same reason only the well-established four-star resonances are included into calculation, namely,  $P_{11}(1440)$ ,  $S_{11}(1535)$ ,  $S_{11}(1650)$ ,  $P_{13}(1720)$ ,  $P_{33}(1232)$ ,  $S_{31}(1620)$ , and  $P_{31}(1910)$ . For the  $\eta$  meson production we also consider the  $P_{11}(1710)$  resonance, because it can give a noticeable contribution to the  $\pi^- p \rightarrow \eta n$  amplitude [1, 16]. The data on the total  $\pi^- p \rightarrow \eta n$  cross section are also

used in the fit. Unfortunately, these experimental data do not agree well with each other, that is why the arguments of [17] were used to clean the database.

## 5 Results and discussion

It was found that the best fit leads to the mixing parameter  $X_{\pi N^*} = 0$  for all resonances. This means the pure pseudovector  $\pi N^*$  coupling. For  $\eta N^*$  coupling the situation is indefinite to some extent, but nevertheless the pseudovector coupling gives slightly better description of data. Only for the  $\pi N N$  coupling we found the small, but nonzero, value  $X_{\pi N} = 0.1$ . The possible explanation of this phenomenon is the PS-PV equivalence theorem, which is valid when all particles entering the vertex are on the mass shell. The difference of the PS and PV couplings, that is mixing effect, appears due to the energy-dependent behaviour of the resonance width. This difference can be detected in the energy region far from the resonance. But in this region the resonance contribution to the amplitude becomes small as compared to others. The situation is different for the nucleon only that is mainly due to the large  $\pi N N$  coupling constant. Taking this fact into account, we put  $X_{\pi N^*} = 0$  and  $X_{\eta N^*} = 0$  in further calculations to decrease the number of free parameters. In Fig. 1 we show the energy dependence of the partial amplitudes for  $S$  and  $P$  spin-isospin channels. The agreement of our  $\pi N$  interaction model (solid line) with the results of phase-shift

**Table 1.** Parameters of the model. The  $G$  values (31) are given in  $\text{GeV}^{-2}$ , masses and widths – in MeV. For  $P_{11}(1440) \rightarrow \eta N$  the coupling constant is presented

Parameter	Value	Error	Parameter	Value	Error
$G_\rho^V$	35.3	1.0	$M S_{11}(1650)$	1688.0	6.0
$G_{\sigma\pi}$	20.3	0.7	$\Gamma_{el} S_{11}(1650)$	249.0	18.0
$\kappa$	3.42	0.10	$\Gamma_\eta S_{11}(1650)$	32.0	5.0
$G_{\pi NN}$	13.6	0.1	$\Gamma_{\pi\pi} S_{11}(1650)$	53.0	5.0
$X_{\pi N}$	0.109	0.006	$M P_{11}(1710)$	1743.0	12.0
$G_{\pi N P_{33}}$	28.6	0.2	$\Gamma_{el} P_{11}(1710)$	1.0	0.8
$Z_{P_{33}}$	-0.229	0.006	$\Gamma_\eta P_{11}(1710)$	277.0	36.0
$G_{\sigma\eta}$	-33.0	10.0	$\Gamma_{\pi\pi} P_{11}(1710)$	196.0	21.0
$G_{a_0}$	20.0	2.0	$M P_{13}(1720)$	1754.0	10.0
$M P_{11}(1440)$	1468.0	10.0	$\Gamma_{el} P_{13}(1720)$	65.0	19.0
$\Gamma_{el} P_{11}(1440)$	248.0	18.0	$\Gamma_{\pi\pi} P_{13}(1720)$	292.0	78.0
$G_\eta P_{11}(1440)$	1.63	0.20	$Z P_{13}(1720)$	-0.50	0.05
$\Gamma_{\pi\pi} P_{11}(1440)$	173.0	14.0	$M S_{31}(1620)$	1627.0	6.0
$M S_{11}(1535)$	1548.0	8.0	$\Gamma_{el} S_{31}(1620)$	75.0	3.0
$\Gamma_{el} S_{11}(1535)$	102.0	8.0	$\Gamma_{\pi\pi} S_{31}(1620)$	83.0	6.0
$\Gamma_\eta S_{11}(1535)$	173.0	20.0	$M P_{31}(1910)$	2060.0	57.0
$\Gamma_{\pi\pi} S_{11}(1535)$	9.1	2.0	$\Gamma_{el} P_{31}(1910)$	40.0	15.0
			$\Gamma_{\pi\pi} P_{31}(1910)$	235.0	58.0

analyses [14, 15] is good up to  $P_{lab} \sim 1$  GeV/c. Small deviation is seen for the  $S_{11}$  amplitude in the region  $P_{lab} \sim 800$  MeV/c. In this region, the contributions from  $S_{11}(1535)$  and  $S_{11}(1650)$  resonances cancel each other, hence the amplitude is very sensitive to small details. It is interesting to note that our results reproduce much better the previous (SM90) version of Arndt's phase-shift analysis in this region (see [14]). Some discrepancy in the  $P_{31}$  partial wave for  $P_{lab} > 750$  MeV/c can be explained by the non-resonance part of the two pion production amplitude, which is not taken into account in the present work.

In the table the parameters of the model are displayed. For the resonances, the mass and partial widths were taken as free parameters, and only for  $P_{11}$  decaying into  $\eta N$  the corresponding coupling constant is available. Other parameters are defined as follows:

$$G_\rho^V = \frac{g_{\pi\rho}^V g_{\rho NN}}{m_\rho^2}, \quad G_{\sigma\pi} = \frac{g_{\pi\sigma} g_{\sigma NN}}{m_\sigma^2}, \quad (31)$$

$$G_{\sigma\eta} = \frac{g_{\eta\sigma} g_{\sigma NN}}{m_\sigma^2}, \quad G_{a_0} = \frac{g_{a_0\pi\eta} g_{a_0 NN}}{m_{a_0}^2}, \quad \kappa = \frac{g_{\rho NN}^T}{g_{\rho NN}^V}.$$

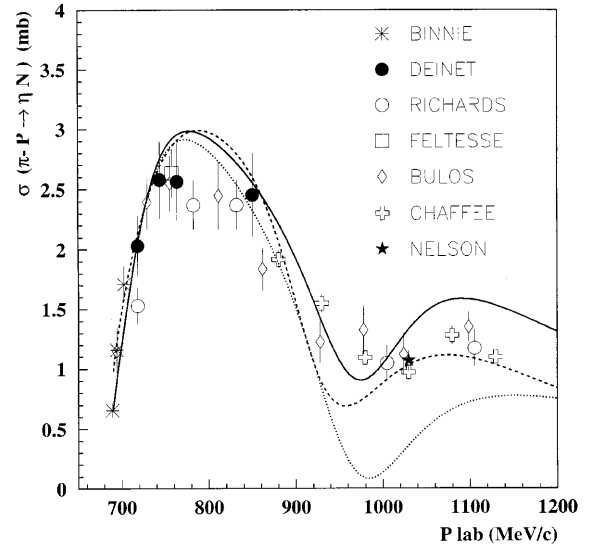
For the non-resonance interaction, we found the values of the parameters close to those of [5] (for  $G_\rho^V = 30.5$   $\text{GeV}^{-2}$ ). The  $\pi N$  scattering lengths calculated with these parameters are  $a_1 = 0.151 m_\pi^{-1}$  and  $a_3 = -0.091 m_\pi^{-1}$ , in qualitative agreement with recent data [5]. The resonance parameters are consistent with the values presented in PDG [1]. It should be noted that only four-star resonances are considered in the present work. Therefore, a possible existence of other resonances [1, 14] can change the values of the parameters shown in Table 1. To obtain more reliable results, a detailed analysis of the experimental data (cross sections, polarization and so on) is needed.

To estimate how the results depend on phase shifts analysis we have also fitted the KH80 solution. The conclusions drawn from this fit are:

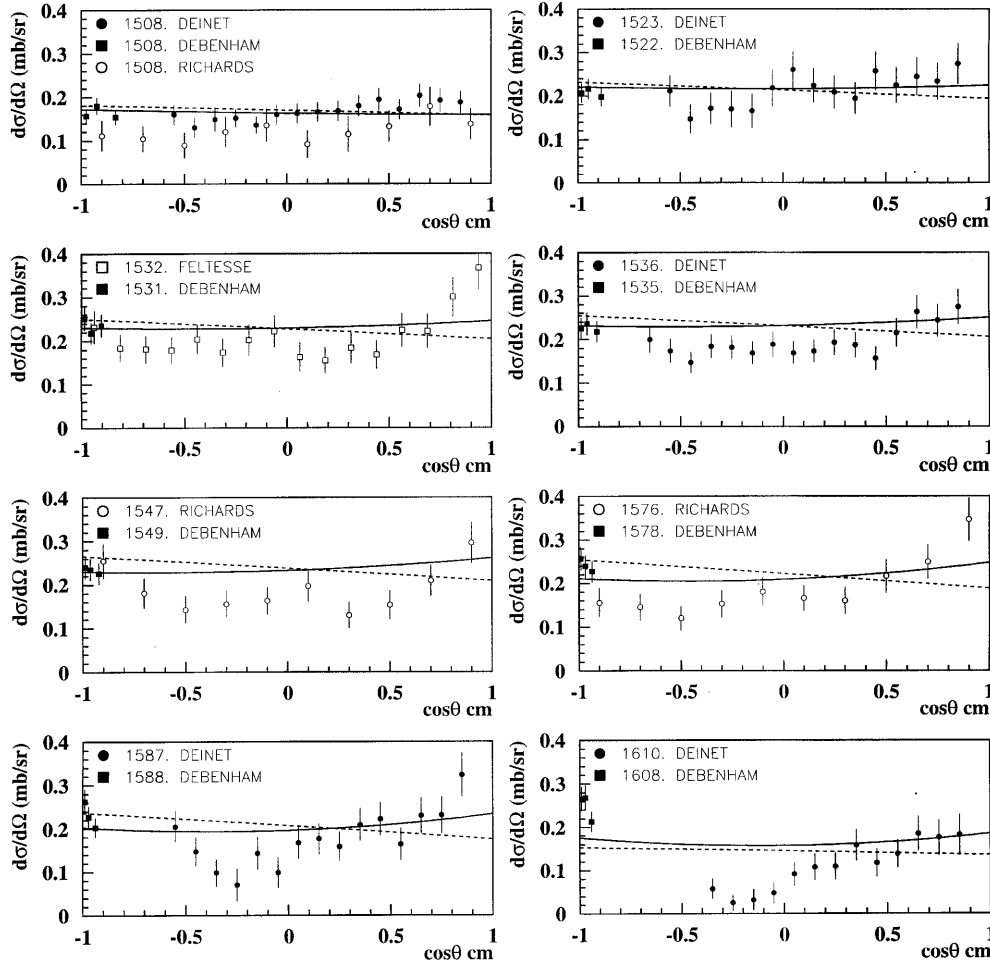
- None of the model parameters are beyond the limits (unfortunately large) published in [1].
- It is rather difficult to describe simultaneously both the  $S_{11}$  amplitude in the region  $550 < P_{lab} < 750$  MeV/c and the energy dependence of the total cross section of the  $\pi^- p \rightarrow \eta n$  reaction at the threshold.

We found that including the  $t$ -channel graph with the  $a_0$  meson exchange leads to a significant improvement ( $\approx 10\%$ ) in  $\chi^2$ . At the same time, the contribution of the  $\eta$ -meson production diagram with nucleon in the intermediate state becomes very small and can be neglected. This graph is usually used to describe the  $\eta$ -meson photoproduction cross section [4]. On the other hand, in the fit without the  $a_0$  exchange (dashed line in Fig. 1) we can determine the contribution of this nucleon Born term with parameters close to those used in the  $\eta$ -meson photoproduction: the coupling is pseudoscalar, with the constant  $g_{\eta NN} = 0.4$ . Therefore, it is possible that not all the  $t$ -channel diagrams are accounted for in the  $\eta$ -meson photoproduction. For example, the  $\phi(1020)$  meson has a large  $\gamma\eta$  branching ratio 1.26% as compared to the  $\rho$  and  $\omega$  mesons, whose  $\gamma\eta$  branching ratios are  $\sim 10^{-4}$ . But this meson has not been included in the analysis yet. In any case the simultaneous analysis of the pionic and photo  $\eta$ -meson production is needed to understand the role of non-resonance processes.

In Fig. 2 the energy dependence of the total cross section of  $\pi^- p \rightarrow \eta n$  reaction is shown. Solid line corresponds to the full calculation and dashed one stands for the fit without the  $a_0$ -meson exchange diagram. As is seen from



**Fig. 2.** Total cross section of  $\pi^- p \rightarrow \eta n$  reaction. Solid line corresponds to full calculation, dashed line stands for calculation without  $a_0$  exchange, dotted line stands for S wave contribution to the  $\sigma^{tot}$ . Experimental data are from [18, 19, 20, 21, 22, 23, 24]



**Fig. 3.** Differential cross section of  $\pi^-p \rightarrow \eta n$  reaction. Solid line corresponds to full calculation, dashed line stands for calculation without  $a_0$  exchange, Experimental data are from [19, 20, 25]

the figure, the errors of the experimental data are large and, therefore, these data are not sensitive to the details of the mechanism. The peak at  $P_{lab} \approx 760$  MeV/c roughly corresponds to the first  $S_{11}$  (1535)-resonance position, whereas the second maximum at  $P_{lab} \approx 1100$  MeV/c is usually explained by the P-wave contribution to  $\sigma^{tot}$ . Here we would like to note that such a structure may be in the S-wave only, due to the interference of two  $S_{11}$  resonances. Indeed, the S wave  $K$ -matrix for the  $\pi p \rightarrow \eta n$  transtion in this region has the following form:

$$\frac{\alpha_{1\pi}\alpha_{1\eta}}{M_1 - \omega} + \frac{\alpha_{2\pi}\alpha_{2\eta}}{M_2 - \omega} + \text{non-resonance terms.} \quad (32)$$

It is seen from this formula that there exists an energy  $M_1 < \omega_0 < M_2$ , where  $K_{\pi \rightarrow \eta}(\omega_0) = 0$ . Since the second  $S_{11}$  (1650) resonance has a small branching ratio for the  $\eta$  decay, this energy must be close to  $M_2$  ( $P_{lab} \approx 980$  MeV/c). To demonstrate this, in Fig. 2 are shown the calculation results, where the S wave is kept in  $\sigma_{\pi \rightarrow \eta}^{tot}$  only (dotted line). The minimum at  $P_{lab} = 980$  MeV/c is clearly seen. In Fig. 3 the differential cross sections for the  $\pi^-p \rightarrow \eta n$  reaction are presented. Unfortunately, large

errors and some contributions in the data do not permit to extract the P-waves contribution reliably. Probably the best way is to measure the  $\eta$  meson production on a polarized target. Indeed, the polarization parameter  $P$  is defined as follows:

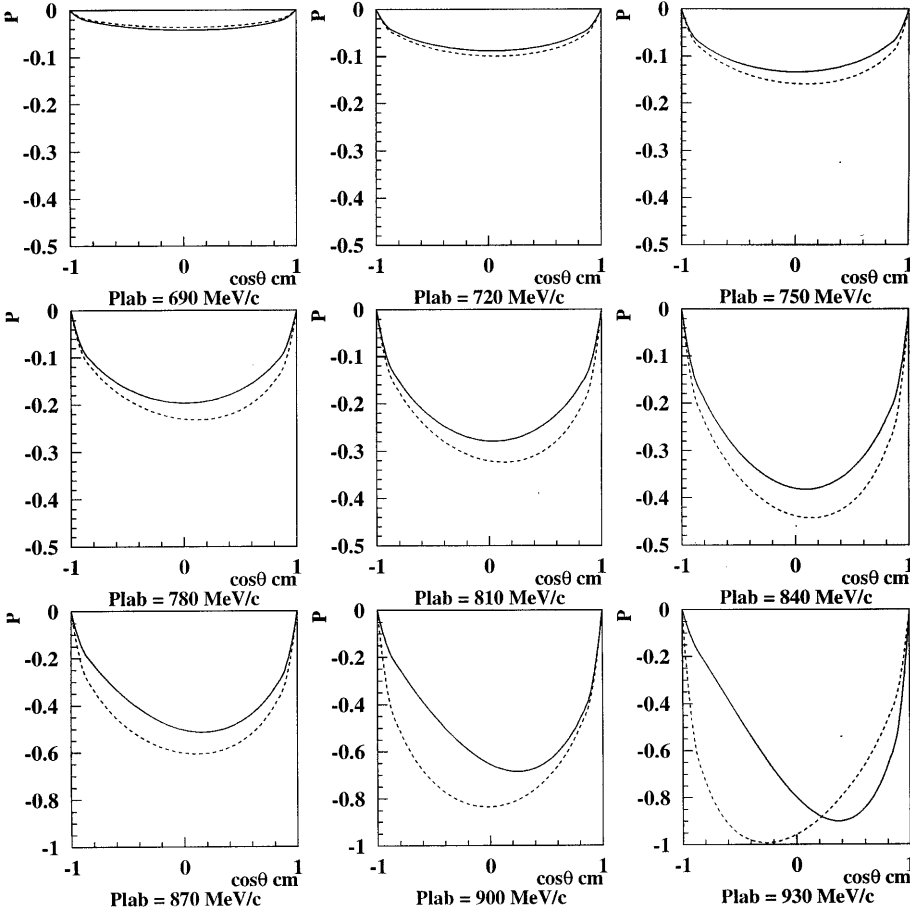
$$P = \frac{2Im[GH^*]}{|G|^2 + |H|^2}, \quad (33)$$

where  $G$  and  $H$  are the non-spin-flip and spin-flip invariant amplitudes for  $\pi^-p \rightarrow \eta n$  reaction. These amplitudes can be expressed in terms of the partial-wave amplitudes  $F_{l\pm}$  according to definitions:

$$G = \sum_{l=0}^{\infty} [(l+1)F_{l+} + lF_{l-}] P_l(\cos\theta)$$

$$H = \sin\theta \sum_{l=1}^{\infty} [F_{l+} - lF_{l-}] P_l'(\cos\theta). \quad (34)$$

The expansion of the  $H$  amplitude starts with  $l = 1$ , hence the polarization parameter  $P = 0$  for S waves only. To show the order of the magnitude for possible measure-



**Fig. 4.** Polarization parameter  $P$  in the  $\pi^-p \rightarrow \eta n$  reaction. Solid line corresponds to full calculation, dashed line stands for calculation without  $a_0$  exchange

ments, in Fig. 4 the calculated values of  $P$  are presented for several energies.

## 6 Conclusions

- The  $K$ -matrix approach gives a natural way to separate the resonance and non-resonance pieces of the  $\pi N$  amplitude.
- The tree level model for the  $K$ -matrix with effective Lagrangians leads to a good description of all S and P elastic partial amplitudes without introducing any phenomenological form factors. To understand remaining difference, a more sophisticated theoretical analysis is needed.
- For the inelastic  $\pi p \rightarrow \eta n$  reaction, a satisfactory agreement with available experimental data is observed. The  $t$ -channel  $a_0$  meson exchange mechanism leads to a considerable improvement of the overall description of data. But to specify reliably the  $\pi^-p \rightarrow \eta n$  reaction mechanism (in particular the P and D wave contributions), more accurate experimental data are needed together with a refined theoretical study. The measurement of the polarization parameter  $P$  is proposed.
- To determine carefully the resonance parameters, an explicit analysis of the experimental data (cross sections and polarization parameters) is needed. The study of all possible (not only four-star) resonances is necessary.

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## Appendix

The  $K$ -matrix for the process with the resonance which can decay into  $n$  different channels has a form:

$$K = \begin{pmatrix} \frac{\alpha_1 \alpha_1}{M-w} + \beta_{11} & \frac{\alpha_1 \alpha_2}{M-w} + \beta_{12} & \dots & \frac{\alpha_1 \alpha_n}{M-w} + \beta_{1n} \\ \frac{\alpha_2 \alpha_1}{M-w} + \beta_{21} & \frac{\alpha_2 \alpha_2}{M-w} + \beta_{22} & \dots & \frac{\alpha_2 \alpha_n}{M-w} + \beta_{2n} \\ \vdots & \dots & \ddots & \vdots \\ \frac{\alpha_n \alpha_1}{M-w} + \beta_{n1} & \frac{\alpha_n \alpha_2}{M-w} + \beta_{n2} & \dots & \frac{\alpha_n \alpha_n}{M-w} + \beta_{nn} \end{pmatrix}, \quad (\text{A.1})$$

where  $\beta_{ij}$  is corresponding non-resonance term. Let us prove that at  $w = M$   $\det K$  can contain the first order pole only. From the determinant relation, we get:

$$\begin{aligned}
& \det K \\
&= \det \begin{pmatrix} \frac{\alpha_1 \alpha_1}{M-w} & \frac{\alpha_1 \alpha_2}{M-w} & \cdots & \frac{\alpha_1 \alpha_n}{M-w} \\ \frac{\alpha_2 \alpha_1}{M-w} + \beta_{21} & \frac{\alpha_2 \alpha_2}{M-w} + \beta_{22} & \cdots & \frac{\alpha_2 \alpha_n}{M-w} + \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_n \alpha_1}{M-w} + \beta_{n1} & \frac{\alpha_n \alpha_2}{M-w} + \beta_{n2} & \cdots & \frac{\alpha_n \alpha_n}{M-w} + \beta_{nn} \end{pmatrix} \\
&+ \det \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \frac{\alpha_2 \alpha_1}{M-w} + \beta_{21} & \frac{\alpha_2 \alpha_2}{M-w} + \beta_{22} & \cdots & \frac{\alpha_2 \alpha_n}{M-w} + \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_n \alpha_1}{M-w} + \beta_{n1} & \frac{\alpha_n \alpha_2}{M-w} + \beta_{n2} & \cdots & \frac{\alpha_n \alpha_n}{M-w} + \beta_{nn} \end{pmatrix}.
\end{aligned} \tag{A.2}$$

The first determinant in (A.2) can be expressed by:

$$\begin{aligned}
& \det \begin{pmatrix} \frac{\alpha_1 \alpha_1}{M-w} & \frac{\alpha_1 \alpha_2}{M-w} & \cdots & \frac{\alpha_1 \alpha_n}{M-w} \\ \frac{\alpha_2 \alpha_1}{M-w} & \frac{\alpha_2 \alpha_2}{M-w} & \cdots & \frac{\alpha_2 \alpha_n}{M-w} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_n \alpha_1}{M-w} + \beta_{n1} & \frac{\alpha_n \alpha_2}{M-w} + \beta_{n2} & \cdots & \frac{\alpha_n \alpha_n}{M-w} + \beta_{nn} \end{pmatrix} \\
&+ \det \begin{pmatrix} \frac{\alpha_1 \alpha_1}{M-w} & \frac{\alpha_1 \alpha_2}{M-w} & \cdots & \frac{\alpha_1 \alpha_n}{M-w} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\alpha_n \alpha_1}{M-w} + \beta_{n1} & \frac{\alpha_n \alpha_2}{M-w} + \beta_{n2} & \cdots & \frac{\alpha_n \alpha_n}{M-w} + \beta_{nn} \end{pmatrix}.
\end{aligned} \tag{A.3}$$

The first term in (A.3) equals to zero, since it contains two proportional rows. Repeating this procedure for the second terms in (A.2) and (A.3), we see that only one row contains the pole in the final expression. Hence  $\det K$  has the first order pole only. Under unitarity transformation (18) the value of  $\det K$  is conserved. But the transformed matrix  $K^e$  is diagonal, therefore  $\det K^e$  is the product of its diagonal terms. So, we conclude that only one element of  $K^e$  can contain the pole term. Another quantity which is conserved under the unitarity transformation (18) is

the trace. So, this pole term has the following form:

$$\sum_i \frac{\alpha_i \alpha_i}{M-w}. \tag{A.4}$$

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